this shows that the basis of a vector space need not be unique.

The vector space V is said to be finite dimensional if there exists a finite subset S of V such that L(S) = V. Otherwise the vector space is infinite dimensional.

The null vector space, which has no basis is of finite dimension, which is zero.

The number of elements in any basis set of a finite dimensional vector space V(F) is called the dimension of the vector space and is denoted by dim V.

$V_n(F)$ is n- dimensional, if its basis contains n elements.

The dimension of the vector space R^n is 2, since $B = \{(1,0), (0,1)\}$ is a basis

the vector space R^3 is of dimension 3, as $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ is a basis of R^3

Similarly for R^4, it is 4.

Let W be a proper Sub-space of V. Any Sub-Set of W containing $(n+1)$ or more vectors is also a Sub-set of V and any $(n+1)$ vectors in V are linearly dependent. Hence any linearly independent set of vectors in W can contain atmost n vectors, let

$$S = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$$

be a linearly independent proper sub-set of W with a maximum number of elements. We shall show that S is a basis of W.

We have assumed that the largest independent set of W contains m vectors; hence (m+1) vectors $\alpha, \alpha_1, \alpha_2, \ldots, \alpha_m$ belonging to W are linearly dependent. Therefore there exists a vector in W which can be expressed as a linear ~~independent.~~ ~~Therefore this vector~~ combination of the preceding vectors. Since $\alpha_1, \alpha_2, \ldots, \alpha_m$ are linearly independent, therefore this vector cannot be any one of these m vectors.

Therefore it must be α. Thus α can be expressed as a linear combination of $\alpha_1, \alpha_2, --, \alpha_m$.

So $L(S) = W$

Therefore S is a basis of W and hence $\dim W = m$ and $m \le n$.

Hence the theorem

Extension theorem

Every linearly independent sub-set of a finitely generated vector space $V(F)$ is either a basis of V or can be extended to form a basis of V.

Proof: Since $V(F)$ is finite dimensional, there exists a basis set of V. Let $S = \{\alpha_1, \alpha_2, --, \alpha_m\}$ be a linearly independent sub-set of a finite dimensional vector space $V(F)$. If $\dim V = n$, then V has a finite basis and let it be $\{\beta_1, \beta_2, --, \beta_n\}$

Consider the set $S_1 = \{\alpha_1, \alpha_2, --, \alpha_m, \beta_1, \beta_2, --\beta_n\}$ so that $V = L(S_1)$

Since the α's can be expressed as a linear combination of the β's, hence the set S_1 is linearly dependent. Thus there is some vector S_1 which is a linear combination of the preceding vectors. Since the α's are linearly independent, this vector can not be any one of the α's. It must be, therefore, some of the β's, say β_i. Now exclude the vector β_i from S_2 and consider the set

$$S_2 = \{\alpha_1, \alpha_2, \ldots, \alpha_m, \beta_1, \beta_2, \ldots \beta_{i-1}, \beta_{i+1}, \ldots, \beta_n\}$$

obviously, $V = L(S_2)$

If the set of vectors S_2 be linearly independent, then S_2 is a basis of V and it is the required extended set which is a basis of V. If S_2 be not linearly independent, then repeating the above process a finite number of times, one can get a linearly independent set which will contain $\alpha_1, \alpha_2, \ldots, \alpha_m$ and will span V. This set will be a basis of V and will contain S. Since each basis of V contains the same number of elements. Hence exactly $(n-m)$ elements of the set β's will be adjoined to S so as to form basis of V.

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		
M	T	W	T	F	S	S

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dependent set, at least one of the vectors of the set, say α_j, can be expressed as a linearly combination of the remaining others.

Let $\alpha_j = d_1\alpha_1 + d_2\alpha_2 + \dots + d_{j-1}\alpha_{j-1} + d_{j+1}\alpha_{j+1} + \dots + d_n\alpha_n$

for some scalars $d_i \in F$,

$i = 1, 2, \dots j-1, j+1, \dots n$. ---(1)